

# Depreciation

# Systems

FRANK K. WOLF

W. CHESTER FITCH



IOWA STATE UNIVERSITY PRESS / AMES

Docket No. E-7, Sub 1146

OFFICIAL COPY

Apr 27 2018

# Contents

PREFACE, vii

- 1** Financial Aspects of Accounting for Depreciation, 3
- 2** Data, 14
- 3** Survivor Curves, 21
- 4** Salvage Concepts, 51
- 5** Depreciation Systems, 69
- 6** Continuous Property Groups, 139
- 7** Defining Depreciation Systems, 175
- 8** Actuarial Methods of Developing Life Tables, 179
- 9** Renewals, 200
- 10** Pricing Retirements, 210
- 11** Analysis of Unaged Data, 217
- 12** Aging Balances, 251

**Frank K. Wolf, Ph.D., P.E.**, is professor and past chair of the Department of Industrial Engineering at Western Michigan University where he teaches courses in operations research and statistics. He is vice president of Depreciation Programs, Inc., and has developed and presented specialized training programs and seminars in depreciation since 1971.

**W. Chester Fitch, Ph.D., P.E.**, is dean of engineering, emeritus, Western Michigan University. He is retired after more than 40 years of conducting depreciation studies and educating and training depreciation staff. He founded a series of programs providing specialized training in depreciation in 1969 and is currently president of Depreciation Programs, Inc.

© 1994 Iowa State University Press, Ames, Iowa 50014  
All rights reserved

Authorization to photocopy items for internal or personal use, or the internal or personal use of specific clients, is granted by Iowa State University Press, provided that the base fee of \$.10 per copy is paid directly to the Copyright Clearance Center, 27 Congress Street, Salem, MA 01970. For those organizations that have been granted a photocopy license by CCC, a separate system of payments has been arranged. The fee code for users of the Transactional Reporting Service is 0-8138-2457-5/94 \$.10.

Printed on acid-free paper in the United States of America

First edition, 1994

Library of Congress Cataloging-in-Publication Data

Wolf, Frank K.  
Depreciation systems / Frank K. Wolf, W. Chester Fitch.—1st ed.  
p. cm.

Includes bibliographical references and index.

ISBN 0-8138-2457-5 (alk. paper)

1. Depreciation. 2. Public utilities—United States—Accounting. 3. Economic life of fixed assets.

I. Fitch, W. Chester. II. Title.

HF5681.D5W65 1994

657'.73—dc20

93-47634

If these parameters remain constant, then the estimate of the average life will be correct.

Given the proper combination of parameters, it is possible for the Gompertz-Makeham equation to turn upward, so that the number of survivors increases with age and, possibly, exceeds 100%. If the curve exceeds 100% for a short period, then turns downward, the common solution is to set all points greater than 100% to 100%. If the curve takes in a continuous, upward trend, the parameters must be adjusted.

## NOTES

1. This treatment assumes retirements occur uniformly during the final age interval and that the life of the longest lived unit is  $ML + .5$  years. If the life of the longest lived unit is known, this value can be used as the end of the final age interval and can be used to calculate the midpoint of the final interval.

2. The maximum life for this Iowa R2 curve is 9.0 years. If the final age interval had been defined to be 8.5 to 9.0 years, the corresponding area would have been 0.21 percent-years, and the total area would have been 500.00 percent-years. See note 1.

# 4

## Salvage Concepts

**S**ALVAGE can be divided into two components: gross salvage and cost of retiring. *Gross salvage* is the value of a unit retired from service resulting from its sale for scrap or reuse. *Cost of retiring*, also called cost of removal, is the expense incurred to remove the unit from service, including expenses necessary to return the environment to an acceptable condition. Thus, *net salvage* is the gross salvage less the cost of retiring.

The original cost less net salvage is called the *depreciable base*. It represents the capital consumed during the life of the unit and the amount to be recovered through depreciation. If the net salvage is positive, then the capital consumed is less than the original cost. If the net salvage is negative, the capital consumed is greater than the original cost.

When net salvage is zero or near zero, its effect on the depreciable base is nil. However, industrial property exhibits a wide range of salvage, and the effect of salvage on the annual accrual is often substantial. Examples of property yielding positive salvage include land, which is generally assumed to be fully recoverable; buildings and vehicles, which often have significant resale value; and aluminum or copper wire, which has a gross salvage value determined by the intrinsic value of the material. Utility poles and railroad track are often reused, and if the accounting system defines a unit as retired when it is removed from a location, its salvage is determined by its value when it is installed in a new location. On the other hand, underground pipe used for transportation or distribution of gas or water must first be disconnected and then may be filled and capped, or even removed from the ground. These activities are costly because they require significant labor

and heavy equipment, while the gross salvage is nil or negligible. The result is a net salvage that is often both large and negative. Decommissioning costs of a nuclear generating plant are a contemporary example of an investment with a significant negative net salvage.

Basic salvage concepts must be understood before either the analysis of realized salvage or the forecasting of future salvage can be discussed. Most of these concepts can be applied equally well to either gross salvage or cost of removal, so the term *salvage* is used generically to apply to net salvage, gross salvage, or cost of retiring.

Property placed in service during the same year forms a *vintage group*. The fraction of the vintage group remaining in service is a function of its age and is described by a survivor curve. An underlying functional relationship between the age at retirement and salvage is assumed. A formal development of how salvage changes as property ages is necessary to understand the effect of salvage on depreciation.

A salvage curve is the graph of the salvage ratio versus age. The salvage ratio is the ratio of the salvage to the original cost of the retired unit. The salvage received during any age interval is found by multiplying the salvage ratio for that interval by the dollars retired during that interval. The net salvage ratio is the gross salvage ratio less the cost of retiring salvage ratio.

As one example of a salvage curve, consider property that is easily removed from service and is still functional after retirement. Gross salvage of early retirements will be high if the property is in good condition and the technology is current, because the property will be valuable for sale or reuse. Older retirements would be less valuable because, besides their added wear, they would be competing for use with property that has a more current technology. If the cost of retiring is assumed to be near zero, this model would lead to a net salvage schedule where the salvage ratio is initially near one, but then decreases with age. This example could be expanded to include retirements resulting from damage from an accident or mechanical failure. Because of their physical condition, these units would have a salvage ratio near zero and would lower the overall salvage ratio.

A salvage curve need not decrease with age. The gross salvage of scrap copper, steel, or aluminum typically, because of inflation, increases with age. A cost of retiring that is labor and equipment intensive is another example of a salvage curve that, because of inflation, increases with age. Because this element of salvage is a cost, the term "increases with age" means the salvage becomes more negative with age. Retirement of a utility pole is an example of an activity for which the hours required to remove the pole might remain relatively constant, but the hourly labor rate, and therefore the cost of retirement, would increase as the pole ages.

There are three reasons why it is important to consider salvage as a

function of age, rather than simply using an overall average salvage. First, though the average life (AL) procedure uses an accrual rate based on the average net salvage, the equal life group (ELG) procedure uses the net salvage associated with each equal life group (i.e., salvage by age). Second, the calculated accumulated depreciation (CAD) model must reflect the change in salvage with age if it is to approximate the accumulated provision for depreciation. Because the CAD is the feedback measure used to determine the adequacy of the accumulated provision for depreciation, it is important that the model used be as lifelike as possible. When the remaining life method of adjustment is used, the amount to be recovered is found by adjusting for the future salvage. These first two reasons show that regardless of the system of depreciation used, both the average and the future salvage are required. Finally, considering salvage as a function of age results in a more realistic model and therefore enhances understanding of the depreciation process and aids in forecasting.

## THE SALVAGE RATIO

One inherent characteristic of the salvage ratio is that the numerator and denominator are measured in different units; the numerator is measured in dollars at the time of retirement, while the denominator is measured in dollars at the time of installation. Inflation is an economic fact of life and although both numerator and denominator are measured in dollars, the timing of the cash flows reflects different price levels. Consider the pattern of installations and retirements illustrated in Figure 4.1 (see end of chapter).

Two replacement cycles are represented. The installation cost of the first unit is  $B$  dollars, it lasts  $K$  years, and has a net salvage of  $V$  dollars. The salvage ratio of the first unit is  $SR(\text{present}) = V/B$ . If the cost of the replacement when measured in constant dollars is equal to the cost of the first unit, then the replacement cost measured in inflated dollars is  $B \times (1 + p)^K$ . The factor  $(1 + p)^K$  is called the compound amount factor and equals the value of \$1 after  $K$  years when the annual rate of inflation is  $p$ . Suppose the life of the replacement unit is  $L$  years and during its life the annual rate of inflation is  $f$ . Then the future salvage of the replacement is  $V \times (1 + f)^L$ . The salvage ratio of the replacement is  $SR(\text{future}) = V \times (1 + f)^L / B \times (1 + p)^K$ . If the past inflation rate  $p$  equals the future inflation rate  $f$ , and if the life of the original equals that of the replacement, so that  $K$  equals  $L$ , then the two inflation factors will be equal. The salvage ratio for the replacement will equal  $V/B$ , unchanged from the original ratio.

This simple model illustrates two important characteristics of the salvage ratio when the uninflated original cost and uninflated salvage remain

constant. One is that a change in the inflation rate will cause a change in the salvage ratio. The other is that a change in service life will change the salvage ratio.

The magnitude of the change in salvage ratio depends on  $p$ ,  $f$ ,  $K$ , and  $L$ . As an example, assume that the past inflation rate,  $p$ , has been 3% during the past  $K$  years, that  $V/B = 10\%$ , and that the life of the replacement is also  $K$  years. Future salvage ratios are determined by the function  $10\% \times [(1 + f)^K / (1 + p)^K]$ . Table 4.1 (see end of chapter) shows future salvage ratios for different values of  $f$ , the inflation rate during the life of the replacement, and different lives. Notice that if the inflation rate does not change, then the salvage remains unchanged regardless of the life. But if the inflation rate increases, the salvage ratio increases. The longer the life and the greater the change in inflation rate, the more the future salvage ratio deviates from the present 10% ratio. Also note the nonlinear relationship between the salvage ratio and the variables  $f$  and  $K$ .

Table 4.1 uses future inflation rates that are equal to or greater than the inflation rate during the life of the first unit. If a similar table is constructed using future inflation rates that are equal to or less than the inflation rate during the life of the first unit, then the salvage ratios will be equal to or less than the 10% ratio experienced by the first unit.

Inflation does not affect all segments of the economy equally. The cost of construction, capital equipment, and labor can all increase at different rates. Because the cost of retiring is often labor and equipment intensive, this element of salvage may be closely tied to indexes that reflect labor and equipment costs. Gross salvage values may be closely tied to used equipment costs and are likely to inflate at a different rate than the cost of retiring. Allowing for different inflation rates for capital equipment, gross salvage, and cost of retiring requires modification to the model just presented.

Assume the inflation rates affecting the cost of replacing the first unit and the gross salvage are equal and constant during the replacement cycle; call this rate  $h$ . Assume that the cost of retiring inflates at a different rate; call this rate  $j$ . After  $L$  years the net salvage,  $V$ , will equal the (uninflated gross salvage)  $\times (1 + h)^L - (\text{uninflated cost of retiring}) \times (1 + j)^L$ . We can use this model to find how the net salvage ratio is affected when these two inflation rates differ.

As an example, assume that the current gross salvage ratio is 20% and that the current cost of retiring ratio is 10%, so that the net salvage ratio is  $20\% - 10\%$  or 10%. The future net salvage ratio will be the net salvage at the end of the life of the replacement unit divided by the installed cost of the replacement unit, or  $[20\% \times (1 + h)^L - 10\% \times (1 + j)^L] / (1 + h)^L$ . Assume that  $h$  is 3% and that the lives of the initial unit and the replacement unit both equal  $L$  years. Table 4.2 (see end of chapter) shows future

salvage ratios for various values of  $L$  and  $j$ . Notice that as the difference between  $h$  and  $j$  becomes larger, the cost of retiring increases faster than the gross salvage. In our example, the cost of retiring catches and exceeds the gross salvage for the larger values of  $j$  and the longer lives. The result is negative net salvage.

The salvage ratio as a function of age and inflation rate can be modeled using the equation  $(V/B) \times (1 + p)^A$ . Table 4.3 (see end of chapter) shows that if the net salvage at time of installation remains constant except for inflation, the observed salvage ratio will vary significantly with time. For example, if the inflation rate was 6% and the salvage ratio at age zero is equal to 10%, the salvage ratio at age 5 would be 13.38% and by age 20 would have increased to 32.07% simply because of inflation. Because the value of the function  $(1 + p)^A$  increases rapidly as  $A$  becomes large, the factors for a large age (e.g., 40 years) are significantly greater than the 10% initial value.

Recognition of the effect of inflation on salvage will influence the analysis and forecasting of salvage. To find the effect of inflation, it is necessary to understand and calculate the time value of money.

## THE SALVAGE CURVE

A salvage curve has been defined as the graph of the salvage ratio as a function of the life of the property. To calculate the average salvage ratio, or the future average salvage ratio at any age, *both the salvage curve and the survivor curve must be known.*

The net salvage curve is the gross salvage curve less the cost of retiring curve. The method of calculating the average salvage ratio (ASR) is to calculate a weighted average of the salvage ratios for each age interval as shown below.

$$\text{ASR} = E(\text{salvage ratio}) = \sum f(i)g(i) \quad \text{for } i = 1, 2, 3, \dots, ML$$

where  $f(i)$  = the retirement frequency during age interval  $i$  and  $g(i)$  = the salvage ratio during age interval, or the ratio evaluated at the midpoint of interval  $i$ , where the age intervals and indexes  $i$  are defined as

$i$	interval $i$	$x(i)$
0	$0.0 \leq \text{service life} < 0.5$	.25
1	$0.5 \leq \text{service life} < 1.5$	1.00
2	$1.5 \leq \text{service life} < 2.5$	2.00
3	$2.5 \leq \text{service life} < 3.5$	3.00
ML	$ML - .5 \leq \text{service life} < ML + .5$	ML

where  $x(i)$  = the midpoint of age interval  $i$  and  $ML$  = the maximum service life.

The functions  $f(i)$  and  $g(i)$  also can be described as continuous functions and the equation written in integral form, but this offers little computational advantage. Discrete functions and the age intervals defined above are consistent with the methods used to describe service life.

Two more measures of salvage are

$$\begin{aligned} \text{RSR}(i) &= \text{the realized salvage ratio at the start of age interval } i \\ &= \Sigma f(k)g(k) / \Sigma f(k) \quad \text{for } k = 1, 2, 3, \dots, i-1 \\ \text{FSR}(i) &= \text{the future salvage ratio at the start of age interval } i \\ &= \Sigma f(k)g(k) / \Sigma f(k) \quad \text{for } k = i, i+1, i+2, \dots, ML \end{aligned}$$

Suppose that the frequency curve and the salvage curve of a group of property are as shown below. The units are retired at ages 0.25, 1, 2, or 3 years with corresponding salvage ratios of 15%, 10%, 5%, or 0%.

Retirement Frequency Curve	Salvage Ratio Curve
$f(0) = .20$	$g(0) = .15$
$f(1) = .30$	$g(1) = .10$
$f(2) = .40$	$g(2) = .05$
$f(3) = .10$	$g(3) = .00$
Total = 1.00	

The average salvage ratio is then calculated as

$$\begin{aligned} \text{ASR} &= \Sigma f(i)g(i) \quad \text{for } i = 1, 2, 3, 4 \\ &= (.20)(.15) + (.30)(.10) + (.40)(.05) + (.20)(0) = 0.08 \text{ or } 8.0\% \end{aligned}$$

Suppose it is the start of the age interval 1.5 to 2.5 years, so that the index  $i$  equals 2. The realized salvage ratio at age 1.5 years,  $\text{RSR}(2)$ , is determined by salvage realized during the first two age intervals, so that.

$$\begin{aligned} \text{RSR}(2) &= [(.20)(.15) + (.30)(.10)] / [.20 + .30] = 0.12 \text{ or } 12\% \\ \text{FSR}(2) &= [(.40)(.05) + (.10)(.00)] / [.40 + .10] = 0.04 \text{ or } 4\% \end{aligned}$$

Note that the weighted average of the realized and future salvage ratios equals the average salvage ratio:

$$\begin{aligned} \text{Weight for RSR}(2) &= .20 + .30 = .50 \\ \text{Weight for FSR}(2) &= .40 + .10 = .50 \\ \text{Weighted average salvage} &= \text{ASR} = .50 \times 12\% + .50 \times 4\% = 8\%. \end{aligned}$$

Table 4.4 (see end of chapter) shows the salvage calculations for an Iowa R2 curve with a 5-year average life ( $R2-5$ ). Column (c) is the percent retired during the age interval and is found by subtracting successive points on the survivor curve shown in column (b). Column (d) shows the average salvage ratio during the age interval. Note that the salvage ratios in this schedule increase with age.

The salvage observed during the age interval depends on both the salvage per unit and the number of units retired. Column (e) is the product of the salvage ratio and the fraction retired. It equals the salvage during the age interval as a percent of the initial cost. During the age interval 2.5 to 3.5 years, the salvage equals 1.21% of the initial cost. The sum of these amounts is the total salvage over the life of the group expressed as a percent of the initial cost; this is the average salvage ratio, which is 13.46%.

Column (f) is the realized salvage ratio and represents the average that would result if an observer recalculated the average salvage ratio at the start of each age interval or each year. The average salvage at age 2.5 years depends on the salvage during each of the preceding three age intervals. The salvage during these intervals is summed to obtain  $0.11\% + 0.38\% + 0.70\%$  or  $1.19\%$ . This amount must be divided by the fraction retired by that age, or  $1 - 0.8913$  or  $0.1087$ , to obtain  $1.19\% / 0.1087$  or  $10.92\%$ . The realized salvage ratio at the start of the second age interval equals the average during the first age interval. As the age increases, the realized salvage ratio approaches the average salvage ratio. At the end of the final age interval the realized salvage ratio, 13.46%, equals the average salvage ratio.

Column (g) is the future salvage ratio, or salvage expectancy, at the start of each age interval. The future salvage ratio at any age is the average salvage ratio observers would calculate if they recorded the salvage from that time on. At age zero the future salvage ratio and the average salvage ratio are equal because both averages include all future salvage ratios. At age 6.5 years, future salvage depends on the salvage during each of the three remaining age intervals. The salvage during these intervals is summed to obtain  $2.25\% + 1.04\% + 0.14\%$  or  $3.43\%$ . This amount must be divided by the future amount to be retired, which is the fraction in service at age 6.5, or  $22.32\%$ , to obtain  $3.43\% / 0.2232$  or  $15.37\%$ . Because the ratios in this salvage schedule increase with age, the future salvage ratios also increase with age.

At any time, the average of the realized and future salvage ratios will equal the overall average salvage. At age 3.5 years, the weighted average of the realized and future salvage ratios is  $11.40\% \times (1 - .7901) + 14.01\% \times (.7901)$  or  $13.46\%$ . Figure 4.2 (see end of chapter) is a graph of the salvage ratio, future salvage ratio, and realized salvage ratio versus age.

## Salvage Schedule Models

A survivor curve must start at 100% and decrease monotonically to zero, but there are no similar constraints for the salvage schedule. The salvage curve can be either increasing or decreasing and need not be monotonic. It need not start at 100% nor end at 0%. There are, however, several basic models that approximate actual patterns and are therefore useful to the analyst and forecaster. We will describe each first in constant dollars and then add inflation. The curve with inflation represents the salvage curve that would be constructed from observed data. The curve without inflation shows the underlying model and is therefore useful when analyzing salvage data.

The first model is a salvage ratio that, when measured in constant dollars, remains constant. This model could reflect the gross salvage of property whose major value is as scrap so that the gross salvage would equal the intrinsic value of the material. It also could be applied to the cost of retiring when the method of removal remains unchanged with time. Table 4.5 (see end of chapter) shows a salvage curve with ratios equal to 10% at all ages. The survivor curve in column (b) is an Iowa R2-5. The salvage curve is shown in column (c); all ratios are equal to 10%. Column (d) is the product of the fraction retired during the age interval and the salvage ratio shown in column (c), and when these are summed the average salvage is found to be 10%. Because the future salvage is needed when calculating depreciation, the future salvage ratios are shown in column (e).

Columns (f), (g), and (h) contain the inflated curves. The inflated ratio is found by multiplying the corresponding, uninflated ratio by the compound amount factor  $(1 + i)^{\text{AGE}}$  where  $i$  is the inflation rate. The salvage curve for the constant model with inflation increases exponentially with age. The 6% inflation rate increases the average salvage ratio from 10% to 13.46% and the salvage ratio at the maximum life, 9 years, to 16.89%. Figure 4.3 (see end of chapter) is a graph of these salvage ratios both with and without inflation. Remember that the difference between the uninflated and inflated salvage ratios increases with age. If an example using a survivor curve with an average life longer than 5 years had been used, the difference between the two ratios would be even larger.

The second model is one in which the salvage ratio decreases uniformly with age. The linear model shown in Table 4.6 and Figure 4.4 (see end of chapter) starts at 100% at age zero (and averages 97.37% during the first age interval) and ends at 0% at age 9.5 years with a resulting annual decrease of  $100\%/9.5$  or 10.53%. The initial value need not be 100%. Suppose, for example, that 20% of the capitalized cost was installation cost. If the property was removed immediately after installation, installation cost

would be lost and, if the full price of the unit was recovered, the salvage ratio would be 80%.

If the survivor curve is symmetrical, the average salvage ratio for the constant dollars model will be the salvage ratio at the midpoint of the curve, which here is the average of the initial and final salvage ratios. Because the survivor curve is the right modal R2 curve, more weight is given to early retirements and the average salvage is less than 50%.

The linear model with inflation also decreases, but in a nonlinear fashion. The shape of the linear model with inflation depends on slope of the line and the inflation rate. The constant model can be considered a special case of the linear model.

The third model reflects an accelerated rate early in life. This model would be particularly applicable to gross salvage when the value falls rapidly early in life and then decreases more slowly later in life. Property such as automobiles and electronic equipment are examples that might follow this pattern. Several mathematical functions could be used to describe this pattern, but a function similar to that used to calculate sum-of-years-digit depreciation was chosen.

To obtain an accelerated curve, first identify the maximum life,  $ML$ , and then sum the digits  $1 + 2 + 3 + \dots + ML = (ML)(ML + 1)/2 = D$ . Next find the total amount by which the salvage ratio will decrease, which is  $S(0) - S(ML)$ . Then find the numerator of the rate for each age interval  $i$ . For age interval 0 to 0.5 years, this is  $ML/2$ . For all other age intervals it is  $ML - i + 0.5$ . The annual decrease of salvage during age interval  $i$  is the product of the total amount of decrease times  $(ML - i + .5)/D$ .

Table 4.7 (see end of chapter) shows the calculation of the average salvage ratio curve using the accelerated model. The initial salvage ratio,  $S(0)$ , was chosen to equal 100% and the salvage ratio at the maximum life was chosen to equal zero, so that  $S(ML) = 0\%$ . The maximum life of the R2-5 occurs during age interval  $i = 9$ , or during the age interval 8.5 - 9.5 years. The sum of digits 1 through 9 is 45. Column (b) shows the numerator of the rate, which is  $L/2 = 9/2 = 4.5$  for the first interval and  $9 - (i + 0.5)$  thereafter. The numerator decreases by 1 each year and the value during the final age interval is always 0.5. Each year the salvage decreases by an amount equal to the total decrease, 100%, times the weight in column (b) divided by 45. During age interval 2.5 - 3.5 this amount is  $100\% \times (9 - 3 + 0.5)/45$  or 14.44%. Because the salvage at age zero is 100%, the value at the end of the first age interval, column (e), is 100% less the decrease of  $(4.5/45) \times 100\%$  or 10%, or 90%. This amount is carried forward to the start of the next age interval. The average salvage during the age interval is shown in column (f).

Table 4.8 (see end of chapter) shows the salvage ratios that would

result if life characteristics are described by the Iowa R2-5 survivor curve and the salvage shown in Table 4.7 is used; the table also shows the salvage ratios with an inflation rate of 6% applied. Figure 4.5 (see end of chapter) shows the salvage ratios without and with inflation plotted versus age.

### Aged Data

Salvage curves reflecting historical salvage can be constructed from aged retirement data using the same techniques used to develop life tables. Because the forces affecting gross salvage and cost of retiring are often independent, these two costs should be recorded, analyzed, and forecasted separately. The net salvage is obtained by subtracting the cost of retiring from the gross salvage.

The requirements for aged salvage data are similar to the requirements for aged retirement data. As with aged retirement data, aged salvage data can be organized in a matrix with rows designating placement years and columns designating experience years.

Data from two sources are necessary to calculate the salvage curve for a vintage group. One set of data is the total salvage dollars during each experience year for the vintage under consideration. The salvage is either the gross salvage or the cost of retiring, depending on which salvage curve is being developed. The second set of data is the annual dollars retired during each experience year of the vintage under consideration. The salvage ratios are calculated directly from these data. The total salvage during the year depends on both the total number of retirements per year and the salvage per unit. The quotient of the total salvage divided by the original cost of the retirements equals the salvage ratio for that experience year.

The first three rows in Table 4.9 (see end of chapter) show the gross salvage, the cost of retiring, and the dollars retired from a 1982 vintage. Remember that the retirements are measured in original cost dollars (i.e., 1982 dollars), but the gross salvage and cost of retiring are measured in experience-year dollars. The ratios are the salvage dollars divided by the dollars retired for the same year. The survivor curve for this placement group shows about 22% of the property installed in 1982 is still in service at the end of 1988, so the resulting survivor curves do not reflect the complete history of the vintage group.

### Conversion to Constant Dollars

An observed salvage ratio is a ratio of dollars at time  $x + \text{age}$  over dollars at time  $x$ , where  $x$  represents the year in which the property was installed. This ratio of mixed dollars often obscures underlying salvage patterns. For example, in the constant model presented in the previous section, the ratios were uniform only when measured in constant dollars,

and the shape of the inflated, or observed, curve concealed the uniform pattern. The underlying patterns are also concealed in the linear and accelerated models. Conversion of the inflated ratios to ratios of constant, or uninflated, dollars reveals the underlying model and is therefore of value to the analyst.

The examples shown in Tables 4.5 through 4.9 assumed inflation at a constant annual rate of 6%. A more accurate view would be that each year is associated with a unique inflation factor and that the product of the annual factors, rather than an average, should be used in the discounting or adjusting process.

An important question centers on which inflation factor to use. Perhaps the most common index is the consumer price index (CPI), which is familiar because it reflects changes in the weighted price of goods and services used by the typical U.S. consumer. It recognizes that different segments of the economy, (e.g., health care, food, housing, energy) have different rates of inflation and that the result is a weighted average of these.

It is desirable to obtain specialized indexes that reflect the inflation rates in special segments of the economy, and in fact firms specialize in estimating these factors. Different indexes may apply to gross salvage and cost of retiring, and the appropriate index for gross salvage in one account will generally differ from that of another account. Once the historical indexes are obtained, they can be stored in the data base and updated each year.

The matrix containing the salvage dollars can be adjusted to convert all entries to a common year or reference point. Most indexes have a base year at which the index is set to 1, and other years are measured in reference to it.

Table 4.9 contains an example of salvage data. Suppose that during the period 1982 to 1988 the annual inflation rate was 6%. Table 4.10 (see end of chapter) shows the salvage values introduced in Table 4.9 converted to 1982 dollars, so that salvage and original cost are measured at the same price level. The resulting salvage ratios now have the inflation removed. The annual salvage dollars can be converted to 1982 dollars by dividing by the factor  $(1 + .06)^{\text{AGE}}$ . In 1985 the age is 3, and the factor is  $1/(1.06)^3 = 1/1.19$  or 0.840. The observed gross salvage during 1985 was \$768 and the observed cost of retiring was \$329; multiplying by 0.840 yields 1982 price level values of \$645 and \$276 respectively.

The underlying patterns can now be seen more easily. Examine the gross salvage ratio and observe that it is approximately linear and declines by about 6% each year. With inflation removed, the cost of retiring ratio is constant and equals 17%.

A first step in salvage analysis is to convert the observed dollars to constant dollars. Then the constant dollar salvage curves can be examined and fit to a model.

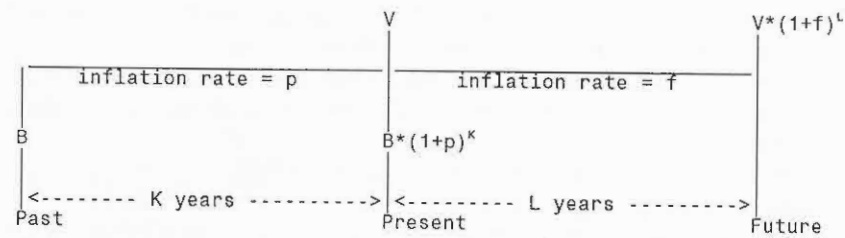


Figure 4.1. A cash flow diagram of investment and salvage costs.

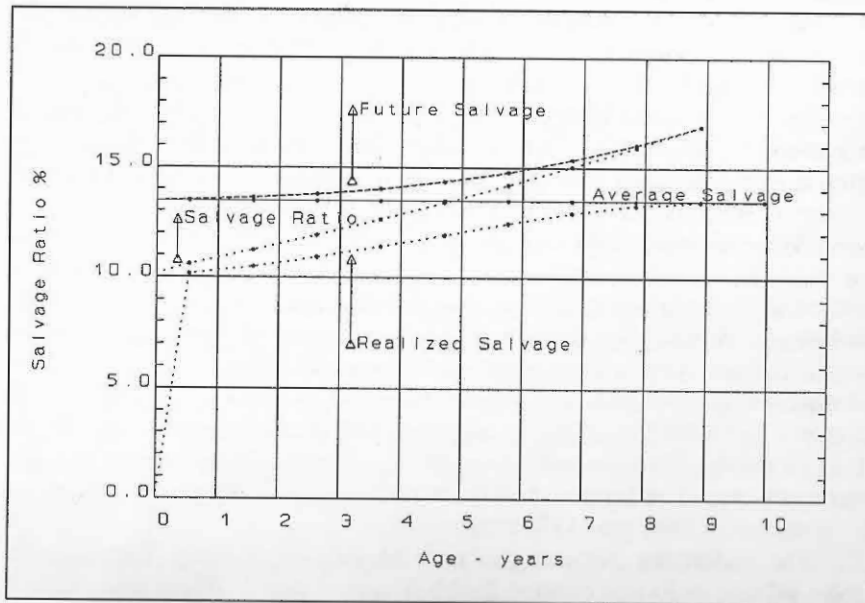


Figure 4.2. A graph of the salvage ratios and the realized and future salvage ratios versus age are for the data shown in Table 4.4.

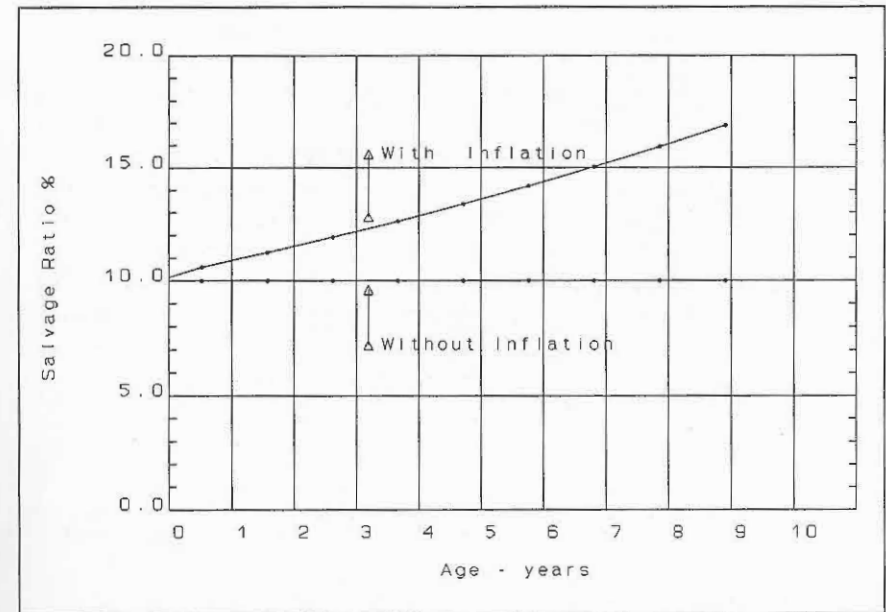


Figure 4.3. A graph of the salvage ratios shown in Table 4.5.

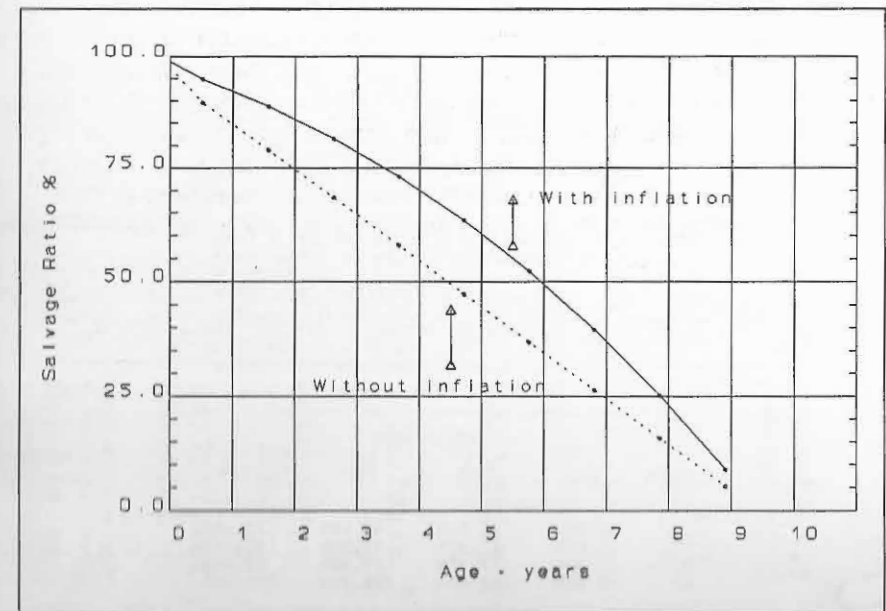


Figure 4.4. A graph of the salvage ratios shown in Table 4.6.

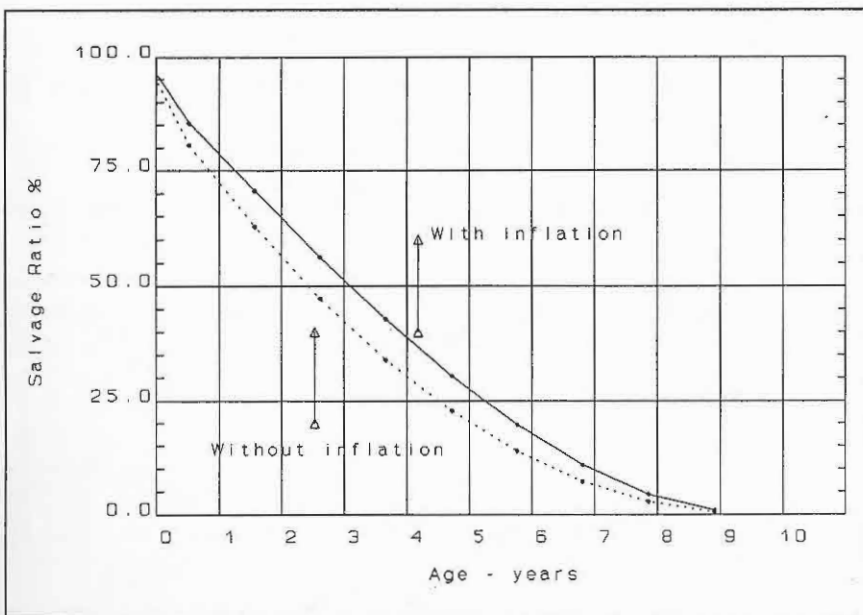


Figure 4.5. A graph of the salvage ratios shown in Table 4.8.

Table 4.1. Future salvage ratios as a function of the future inflation rate,  $f$ , and the life of the unit,  $K$ . The salvage ratio of the first unit is  $V/B = 10\%$  and  $p$ , the annual rate of inflation during its life, is 3%. The life of the first unit equals the life of the replacement unit (i.e.,  $K = L$ ).

K Years	Inflation rate - $f$			
	3%	6%	10%	12%
5	10.00%	11.64%	13.89%	15.20%
10	10.00%	13.33%	19.30%	23.11%
20	10.00%	17.76%	37.25%	53.41%
40	10.00%	31.63%	108.75%	265.25%

Table 4.2. Future salvage ratios as a function of the cost of retiring inflation rate,  $j$ , when  $p = 3\%$ , the current gross salvage = 20% of first cost, and the present cost of retiring = 10% of first cost. The life of the first unit equals the life of the replacement unit.

L Years	Inflation rate - $j$			
	3%	6%	10%	12%
5	10.00%	8.46%	6.11%	4.80%
10	10.00%	6.67%	.70%	-3.11%
20	10.00%	2.24%	-17.25%	-33.41%
40	10.00%	-11.53%	-118.75%	-265.25%

Table 4.3. Salvage ratios as a function of life and inflation rate when  $V/B = 10\%$ .

Life Years	Inflation rate - $p$			
	3%	6%	10%	12%
0	10.00%	10.00%	10.00%	10.00%
5	11.59%	13.38%	16.11%	17.62%
10	13.44%	17.91%	25.94%	31.06%
20	18.06%	32.07%	67.27%	96.46%
40	32.62%	102.86%	452.59%	930.51%

Table 4.4. Calculation of average, realized, and future salvage ratios for the salvage schedule shown in column (d) and with life characteristics described by an Iowa R2-5 survivor curve. The percent surviving, realized salvage ratio, and future salvage ratio are all shown at the start of the age interval.

Constant rate w/inflation							
Age interval (a)	Percent survive (b)	Percent retired (c)	Salvage ratio % (d)	Weighted ratio % (e)	Realized salvage % (f)	Future salvage % (g)	Average salvage % (h)
0- 0.5	100.00	1.11	10.15	.11	.00	13.46	13.46
.5- 1.5	98.89	3.56	10.60	.38	10.15	13.50	13.46
1.5- 2.5	95.33	6.20	11.24	.70	10.49	13.61	13.46
2.5- 3.5	89.13	10.12	11.91	1.21	10.92	13.77	13.46
3.5- 4.5	79.01	15.31	12.62	1.93	11.40	14.01	13.46
4.5- 5.5	63.70	20.30	13.38	2.72	11.91	14.34	13.46
5.5- 6.5	43.40	21.08	14.19	2.99	12.44	14.79	13.46
6.5- 7.5	22.32	14.99	15.04	2.25	12.91	15.37	13.46
7.5- 8.5	7.33	6.51	15.94	1.04	13.26	16.05	13.46
8.5- 9.5	.82	.82	16.89	.14	13.43	16.89	13.46
9.5- 10.5	.00	..	..	..	13.46	..	13.46
Average = 13.46%							

Table 4.5. A salvage curve with a constant rate and 6% inflation.

* Constant rate					* Constant rate w/inflation *			
Age interval (a)	Percent survive (b)	Salvage ratio % (c)	Weighted ratio % (d)	Future salvage ratio % (e)	Salvage ratio % (f)	Weighted ratio % (g)	Future salvage ratio % (h)	
0- 0.5	100.00	10.00	.11	10.00	10.15	.11	13.46	
.5- 1.5	98.89	10.00	.36	10.00	10.60	.38	13.50	
1.5- 2.5	95.33	10.00	.62	10.00	11.24	.70	13.61	
2.5- 3.5	89.13	10.00	1.01	10.00	11.91	1.21	13.77	
3.5- 4.5	79.01	10.00	1.53	10.00	12.62	1.93	14.01	
4.5- 5.5	63.70	10.00	2.03	10.00	13.38	2.72	14.34	
5.5- 6.5	43.40	10.00	2.11	10.00	14.19	2.99	14.79	
6.5- 7.5	22.32	10.00	1.50	10.00	15.04	2.25	15.37	
7.5- 8.5	7.33	10.00	.65	10.00	15.94	1.04	16.05	
8.5- 9.5	.82	10.00	.08	10.00	16.89	.14	16.89	
9.5-10.5	.00	--	--	--	--	--	--	
Average = 10.00%					Average = 13.46%			

Table 4.6. A linear salvage curve starting at 100% and declining to 0%, with 6% inflation.

* Linear rate					* Linear rate w/inflation *			
Age interval (a)	Percent survive (b)	Salvage ratio % (c)	Weighted ratio % (d)	Future salvage ratio % (e)	Salvage ratio % (f)	Weighted ratio % (g)	Future salvage ratio % (h)	
0- 0.5	100.00	97.37	1.08	47.35	98.80	1.10	60.94	
.5- 1.5	98.89	89.47	3.19	46.79	94.84	3.38	60.51	
1.5- 2.5	95.33	78.95	4.89	45.19	88.71	5.50	59.23	
2.5- 3.5	89.13	68.42	6.92	42.84	81.49	8.25	57.18	
3.5- 4.5	79.01	57.89	8.86	39.57	73.09	11.19	54.07	
4.5- 5.5	63.70	47.37	9.62	35.16	63.39	12.87	49.49	
5.5- 6.5	43.40	36.84	7.77	29.45	52.26	11.02	42.99	
6.5- 7.5	22.32	26.32	3.94	22.47	39.57	5.93	34.24	
7.5- 8.5	7.33	15.79	1.03	14.61	25.17	1.64	23.35	
8.5- 9.5	.82	5.26	.04	5.26	8.89	.07	8.89	
9.5-10.5	.00	--	--	--	--	--	--	
Average = 47.35%					Average = 60.94%			

Table 4.7. Calculation of an accelerated salvage curve starting at 100% and declining to 0%.

Age interval (a)	Weight (b)	Change (c)	Salvage at start (d)	Salvage at end (e)	Average salvage (f)
0 - .5	4.5	10.00	100.00	90.00	95.00
.5 - 1.5	8.5	18.89	90.00	71.11	80.56
1.5 - 2.5	7.5	18.67	71.11	54.44	62.78
2.5 - 3.5	6.5	14.44	54.44	40.00	47.22
3.5 - 4.5	5.5	12.22	40.00	27.78	33.89
4.5 - 5.5	4.5	10.00	27.78	17.78	22.78
5.5 - 6.5	3.5	7.78	17.78	10.00	13.89
6.5 - 7.5	2.5	5.56	10.00	4.44	7.22
7.5 - 8.5	1.5	3.33	4.44	1.11	2.78
8.5 - 9.5	.5	1.11	1.11	.00	.67

Table 4.8. A salvage curve with the accelerated model shown in Table 4.7 and with 6% inflation.

* Accelerated rate					* Accelerated rate w/inflation *			
Age interval (a)	Percent survive (b)	Salvage ratio % (c)	Weighted ratio % (d)	Future salvage ratio % (e)	Salvage ratio % (f)	Weighted ratio % (g)	Future salvage ratio % (h)	
0- 0.5	100.00	95.00	1.05	26.60	96.39	1.07	32.99	
.5- 1.5	98.89	80.56	2.87	25.83	85.39	3.04	32.28	
1.5- 2.5	95.33	62.78	3.89	23.79	70.54	4.37	30.29	
2.5- 3.5	89.13	47.22	4.78	21.08	56.24	5.69	27.50	
3.5- 4.5	79.01	33.89	5.19	17.73	42.78	6.55	23.81	
4.5- 5.5	63.70	22.78	4.62	13.85	30.48	6.19	19.25	
5.5- 6.5	43.40	13.89	2.93	9.67	19.70	4.15	14.00	
6.5- 7.5	22.32	7.22	1.08	5.68	10.86	1.63	8.62	
7.5- 8.5	7.33	2.78	.18	2.53	4.43	.29	4.04	
8.5- 9.5	.82	.56	.00	.56	.94	.01	.94	
9.5-10.5	.00	--	--	--	--	--	--	
Average = 26.60%					Average = 32.99%			

Table 4.9. Partial retirement and salvage data from the 1982 vintage group. The upper portion of the table shows the gross salvage dollars, cost of retiring, and dollars retired for a vintage group. The lower portion of the table shows the resulting salvage ratios.

	Experience year						
	82	83	84	85	86	87	88
Gross salvage	94	357	470	768	1053	1191	1021
Cost of retiring	27	113	183	329	552	721	784
Annual retirements	157	627	941	1568	2508	3135	3292
Gross salvage ratio	.60	.57	.50	.49	.42	.38	.31
Cost of retiring ratio	.17	.18	.20	.21	.22	.23	.24
Net salvage ratio	.43	.39	.30	.28	.20	.15	.07

Table 4.10. Conversion of salvage in Table 4.9 to 1982 dollars.

	Experience year						
	82	83	84	85	86	87	88
Gross salvage	94	337	418	645	834	890	720
Cost of retiring	27	106	163	276	437	539	553
Annual retirements	157	627	941	1568	2508	3135	3292
Gross salvage ratio	.60	.54	.44	.41	.33	.28	.22
Cost of retiring ratio	.17	.17	.17	.18	.17	.17	.17
Net salvage ratio	.43	.37	.27	.23	.16	.11	.05

## 5

## Depreciation Systems

## T

HE recovery of capital through depreciation accruals may be thought of as a dynamic system. A system is an arrangement of things that are connected to form a complete organization of integrated parts. The state of the system at any time is defined by current values of the characteristics that define the system. A dynamic system is one where the state of the system depends on the history of the input variables. To define and study a system is to better understand the system so that more efficient methods of control can be designed to accomplish the desired ends.

There are two methods of controlling a system. One is to select an input and wait for the result or final output. If a different output is desired, the input is changed and the new output is obtained. The other method of control is to select an initial input, monitor the process, and when necessary, alter the input to achieve the desired goal. The first method is called an open control loop and the second a closed control loop. A necessary feature of the closed control loop is the feedback resulting from the monitoring of the system. A home heating system is a common and simple example of a dynamic system with a closed feedback loop. The parts of the system are a furnace and a thermostat. The thermostat monitors the room temperature and creates feedback, in the form of electrical signals, when the room temperature rises above or falls below the desired temperature. The electrical signals turn the furnace off or on to achieve the desired goal, a constant, predetermined room temperature.

Think of a depreciation accounting system as a dynamic system controlled with a closed feedback loop. Estimates of life and salvage and the